AE 584: Navigation and Guidance of Aerospace Vehicles

Final Project: Drone Landing on a Loitering Aircraft

Due date: Friday, December 13th, by 11:59pm, on Canvas.

Firm Deadline: No extensions will be granted, no matter what the excuse might be. You are responsible of ensuring that your file is submitted on Canvas within the deadline.

______________________, ______________________
LAST NAME (PRINTED) *FIRST NAME*

HONOR PLEDGE: Copy and SIGN: I understand that this is an individual assignment and not a group exercise. I have neither given nor received aid on this project nor have I concealed any violation of the Honor Code. I have neither borrowed nor shared computer code for simulating my models and creating my plots.

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SIGNATURE

This Should Be The Cover Page Of Your Project

General: In this project, our goal is to step-by-step develop a navigation and guidance system that achieves automated landing of an autonomous unmanned aerial vehicle (drone) onto a fixed-wing aircraft that moves on a circular holding pattern. This project aims to illustrate how to design navigation techniques and navigation-based guidance techniques using linear and feedback-linearization tools.

Honor Code: You are to do your own work. Discussing the project with a friend is fine. Sharing code is not allowed.

Overview

An autonomous drone is tasked to perform environmental monitoring by collecting data such as temperature, humidity, and wind speed using on-board sensors. At some time $t_w > t_0$, the battery level drops below a known safety-critical limit. To recharge, the drone has to land on a fixed-wing aircraft that is loitering nearby. However, the communication of the drone with the fixed-wing aircraft has been lost at some time t_c, where $t_0 < t_c < t_w$. In other words, the drone can not receive information about the states of the aircraft for $t > t_c$, and the only available information to the drone about the states of the aircraft is the history of measurements in the time interval $[t_0, t_c]$. In this project, our goal is to step-by-step develop a complete navigation and guidance protocol for this drone to land on the fixed-wing aircraft using the available information from the on-board sensors so that it can charge its battery and resume its monitoring task.

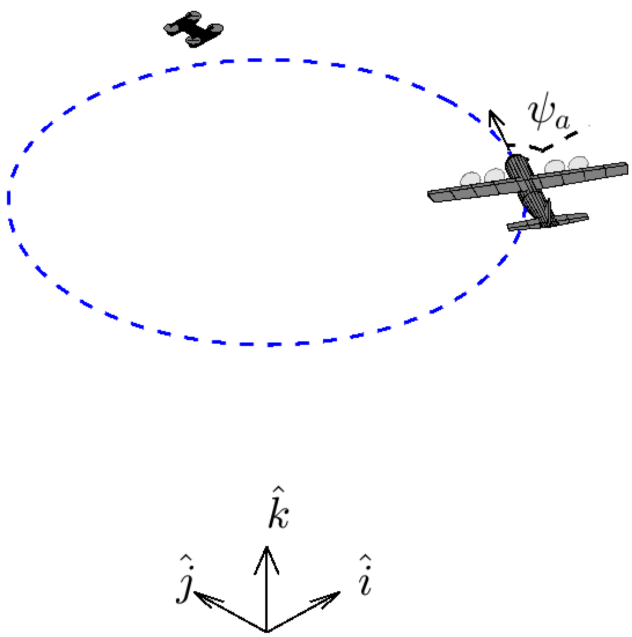
Problem 1 (25 points) *The fixed-wing aircraft is loitering at altitude $z = z_{a0}$. The equations of motion are given as:*

$$\begin{aligned}
\dot{x}_a &= v_a \cos(\psi_a), & x_a(0) &= x_{a0}, \quad &(1a)\\
\dot{y}_a &= v_a \sin(\psi_a), & y_a(0) &= y_{a0}, \quad &(1b)\\
\dot{z}_a &= 0, & z_a(0) &= z_{a0}, \quad &(1c)\\
\dot{\psi}_a &= \frac{v_a}{\rho}, & \psi_a(0) &= \psi_{a0}, \quad &(1d)
\end{aligned}$$

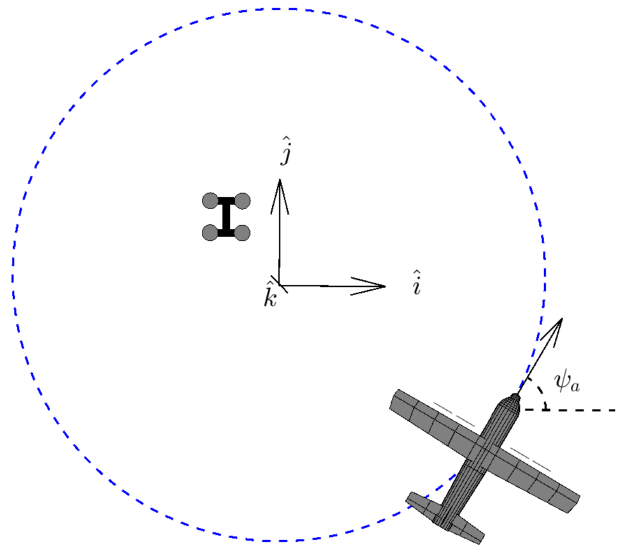
where v_a is the constant speed of the aircraft, and ρ is the constant radius of the circle on which the aircraft is moving. The drone receives the position $(x_a(t), y_a(t), z_a(t))$ and the heading $\psi_a(t)$ of the aircraft at times t that belong to the time interval $[t_0, t_c]$, i.e.,

$$\mathbf{Y}_a(t) = [x_a(t), y_a(t), z_a(t), \psi_a(t)]^T, \quad t \in [t_0, t_c]. \tag{2}$$

Let a set of sensor data obtained at N distinct time instances t_i, $i \in \{1, \dots, N\}$ be denoted as $\mathcal{H} = \Big[\mathbf{Y}_a(t_1), \quad \mathbf{Y}_a(t_2), \quad \dots \quad \mathbf{Y}_a(t_N)\Big]$.



(a) Problem Setup (3D view)



(b) Problem Setup (Top View)

(a) *(5 points) Provide closed-form expressions for the state trajectories of system* (1).

(b) *(10 points) Let us denote the initial conditions in* (1) *as* q_1, q_2, q_3, q_4*, the aircraft speed as* q_5*, and the radius of the circular path as* q_6*. Given a history* $\mathcal{H}$ *of perfect measurements, provide a method to determine the parameters* $(q_1, q_2, ..., q_6)$ *of the aircraft's trajectory.*

(c) *(10 points) In practice, however, sensors provide noisy data. Assume that the measurement vector* $\mathbf{Y}_a(t_i)$, $i \in \{1, \ldots, N\}$*, is corrupted by a zero-mean, additive Gaussian noise vector* $\boldsymbol{\xi} = [\xi_x, \xi_y, \xi_z, \xi_\psi]^T$*, with uncorrelated components whose variances are* $\sigma^2_{\xi_x}, \sigma^2_{\xi_y}, \sigma^2_{\xi_z}, \sigma^2_{\xi_\psi}$*, respectively. Perform an error analysis for the estimates* $(\hat{q}_1, \hat{q}_2, \hat{q}_3, \hat{q}_4, \hat{q}_5, \hat{q}_6)$ *obtained from part (b). In other words, provide the expected value and the covariance matrix of the estimation error, using the history of measurements* $\mathcal{H}$.

Problem 2 (22 points) *You are given measurement histories of the fixed-wing aircraft under the following different scenarios along with the covariances of the respective measurement noises and the true parameters of the trajectories.*

(i) *scenario 1: aircraftData1.mat* ($\sigma^2_{\xi_x} = \sigma^2_{\xi_y} = (2.7)^2$, $\sigma^2_{\xi_z} = (1.2)^2$, $\sigma^2_{\xi_\psi} = (0.02)^2$, $N = 50$)

(ii) *scenario 2: aircraftData2.mat* ($\sigma^2_{\xi_x} = \sigma^2_{\xi_y} = (3)^2$, $\sigma^2_{\xi_z} = (2)^2$, $\sigma^2_{\xi_\psi} = (0.05)^2$, $N = 50$)

(iii) *scenario 3: aircraftData3.mat* ($\sigma^2_{\xi_x} = \sigma^2_{\xi_y} = (2.7)^2$, $\sigma^2_{\xi_z} = (1.2)^2$, $\sigma^2_{\xi_\psi} = (0.02)^2$, $N = 150$)

(a) *(12 points) For each one of the above cases, provide the estimates $(\hat{q}_1, \hat{q}_2, ..., \hat{q}_6)$ of the parameters $(q_1, q_2, ..., q_6)$ of the aircraft trajectories, expected values $(\mu_{\tilde{q}_1}, \mu_{\tilde{q}_2}, ..., \mu_{\tilde{q}_6})$ and the variances $(\sigma^2_{\tilde{q}_1}, \sigma^2_{\tilde{q}_2}, ..., \sigma^2_{\tilde{q}_6})$ of the estimation errors $(\tilde{q}_1, \tilde{q}_2, ..., \tilde{q}_6)$ of the parameters, where $\tilde{q}_i = q_i - \hat{q}_i,\ \forall i \in \{1, 2, ..., 6\}$.*

(b) *(4 points) Plot the x and y position coordinates of the aircraft: 1) on the actual trajectory obtained in Problem 1(a) using the true parameters provided, 2) from the measurement data, and 3) on the estimated trajectory obtained using the mean values of the parameters that you estimated in part (a), for all scenarios. For each scenario, plot the resulting 3 paths on the x-y plane in a single figure. (So, you have to provide a total of three figures in your report).*

(c) *(6 points) Compare and comment on the quality of the estimates for the three cases, i.e., compare the variances of the estimation errors of each case. Compare the plots in part (b). How does the number of measurements N in the history $\mathcal{H}$ affect the estimates?*

Note: Read the Readme.txt file for more details about the data arrangement in the MATLAB data files.

Problem 3 (10 points) *The equations of motion of the drone are given as:*

$$\begin{aligned} \dot{\mathbf{r}} &= \mathbf{v} \\ \dot{\mathbf{v}} &= \mathbf{u} - C_d \mathbf{v} \end{aligned} \tag{3}$$

where $\mathbf{r} = [x, y, z]^T$ and $\mathbf{v} = [v_x, v_y, v_z]^T$ are the position and velocity vectors, and $\mathbf{u}$ is the acceleration input of the drone, all resolved in an inertial frame. C_d is a known drag coefficient. Position measurements that are corrupted by a zero-mean, Gaussian noise $\boldsymbol{\epsilon}$ with covariance matrix $\mathbf{R}_\epsilon$ are available:

$$\mathbf{Y} = \mathbf{r} + \boldsymbol{\epsilon}, \tag{4}$$

where $\boldsymbol{\epsilon} = \mathcal{N}(0, \mathbf{R}_\epsilon)$. Design a (continuous-time) Kalman filter to estimate the full state vector $\mathbf{X} = \begin{bmatrix} \mathbf{r} \\ \mathbf{v} \end{bmatrix}$.

Problem 4 (25 points) *We now consider that the drone needs to approach the aircraft and land on its surface at location $\mathbf{r}_l = [x_a, y_a, z_a + z_l]^T$, where z_l is an offset from the aircraft's center of mass.*

(a) *(15 points) Assuming that the drone has perfect knowledge of its full state vector $\mathbf{X}$, design a state-feedback guidance law using the full state $\mathbf{X}$ and the perfect knowledge of the trajectory of the fixed-wing aircraft obtained in the Problem 1(b), so that the drone approaches the landing point $\mathbf{r}_l$.*

(b) *(10 points) We now recall that the drone has noisy measurements of its position* $\mathbf{r}$*, and that it does not directly measure its velocity* $\mathbf{v}$*. So, consider that the drone uses the estimate* $\hat{\mathbf{X}}$*, instead of the actual value of the state* $\mathbf{X}$ *in the guidance law that you designed in Problem 4(a). Provide the differential equations governing the dynamics of the guidance error* $\delta\mathbf{X} = \mathbf{X} - \begin{bmatrix}\mathbf{r}_l \\ \dot{\mathbf{r}}_l\end{bmatrix}$ *and the estimation error* $\tilde{\mathbf{X}} = \mathbf{X} - \hat{\mathbf{X}}$*.*

Hint: For part (a), you can use the idea of forcing the dynamics of the system follow a desired trajectory by choosing the control action appropriately, as was done in velocity-to-be-gained guidance. Read the chapter 7 in the main textbook (Fundamentals of Aerospace Navigation and Guidance) for more details on this.

Problem 5 (18 points) *(a) (3 points) Simulate the dynamics of the drone and the aircraft by implementing the controller that you designed in Problem 4(b) for the scenario 1 in Problem 2 with the initial condition* $\hat{\mathbf{r}}(0) = [13, 12, 112]^T m$, $\hat{\mathbf{v}}(0) = [0, 0, 0]^T m/s$*. Use the true parameters of the aircraft's trajectory, that are provided in the data files, for this sub-problem. In your report, include the plots for the error in drone's position* $\mathbf{r}$ *and the landing position* $\mathbf{r}_l = [x_a, y_a, z_a + z_l]^T$*, and for the error between drone's velocity* $\mathbf{v}$ *and landing site's velocity* $\dot{\mathbf{r}}_l = \mathbf{v}_a = [\dot{x}_a, \dot{y}_a, \dot{z}_a]^T$*. Use the following numerical data for simulation purpose:* $C_d = 0.1$, $z_l = 4\ m$,

$$\mathbf{R}_\epsilon = \begin{bmatrix}1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.8\end{bmatrix}, \quad \mathbf{P}_{\mathbf{r}(0)} = \begin{bmatrix}0.9 & 0 & 0 \\ 0 & 0.8 & 0 \\ 0 & 0 & 0.5\end{bmatrix}, \quad \mathbf{P}_{\mathbf{v}(0)} = \begin{bmatrix}0.2 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.2\end{bmatrix}, \tag{5}$$

where $\mathbf{P}_{\mathbf{r}(0)}$ *and* $\mathbf{P}_{\mathbf{v}(0)}$ *are the covariances of* $\mathbf{r}(0)$ *and* $\mathbf{v}(0)$*. Assume* $\mathbf{r}(0)$ *and* $\mathbf{v}(0)$ *are jointly Gaussian and uncorrelated.*

(b) *(4 points) Assume that you use the steady-state Kalman gain in your Kalman filter in Problem 5(a); then, what is the steady-state covariance matrix,* $\mathbf{P}_{\delta\mathbf{X}}$*, for the guidance error* $\delta\mathbf{X}$*?*

(c) *(6 points) Now assume that the drone uses the estimated trajectory of the aircraft that you obtained using the parameters you estimated in Problem 2(a), instead of the actual aircraft trajectory information, in the feedback control law you designed. Plot the trajectories of the error* $\mathbf{r} - \mathbf{r}_l$ *and* $\mathbf{v} - \dot{\mathbf{r}}_l$ *for scenario 1 and scenario 2.*

(d) *(5 points) Is the error between the drone's position and the landing position bounded for scenario 1 in Problem 5 (c)? If yes, provide the bound on this error. If not, explain why. How does the error compare in scenario 1 and scenario 2?*

(You can use the animateTraj.m script that is provided on CANVAS for visualization.)

Supporting MATLAB Scripts: (You are free to use your own scripts instead of the provided ones)

- Matlab Script (droneDynamics.m) for calculating the state of the drone after dt time assuming constant input $\mathbf{u}$ during the time interval dt.
- Matlab Script (aircraftDynamics.m) for calculating the state of the aircraft after dt time for the given trajectory parameters.
- A matlab script (animateTraj.m) to visualize the motion of the drone and the aircraft.

Comments are provided in the scripts about their usage.

Write-up for ‘Drone Landing on an Aircraft’ Project (Required Report Format)

You will have many pages of theoretical results, simulations, computations, etc. While most of this should be included in your report, I wish to receive an organized package that is easy to read and follow because I have to go through all of them. Points will be deducted for a poorly presented work.

Page 1: Use the required title page.

Page 2: Contents/Index page. List Problems one through five and include page numbers.

Page 3: Provide answers to the following questions:

A) Provide the estimated aircraft trajectory parameters from the measurement data in Problem 2 in the table format as below.

	scenario 1 (μ, σ^2)	scenario 2 (μ, σ^2)	scenario 3 (μ, σ^2)
$q_1 = x_{a0}$	(__, __)	(__, __)	(__, __)
$q_2 = y_{a0}$	(__, __)	(__, __)	(__, __)
$q_3 = z_{a0}$	(__, __)	(__, __)	(__, __)
$q_4 = \psi_{a0}$	(__, __)	(__, __)	(__, __)
$q_5 = v_a$	(__, __)	(__, __)	(__, __)
$q_6 = \rho$	(__, __)	(__, __)	(__, __)

B) Provide the covariance matrix of the guidance error in scenario 1 as obtained in Problem 5(b): $\mathbf{P}_{\delta\mathbf{X}} =$ __.

Page 4: Begin answers to problems one through five.

- Start each problem on a separate page, starting the page with the number of the problem that is being worked, e.g., Problem 1, etc.
- DO NOT group all simulations and/or hand computations at the end as one giant appendix. Instead, group them with the problem that is being worked. That is, all simulations for Problem 4 will follow Problem 4, those for Problem 5 will be with Problem 5, etc.
- Only include the most important of your printouts.
- Put labels or numbers on your plots. You can number them by hand or using MATLAB.
- Include your MATLAB code at the end of the document, clearly stating the Problem number for which the code was used.
- You do NOT have to typeset your solutions. Handwritten report is perfectly fine.
- The submission is via Canvas. You only have to upload ONE .pdf file, including all the codes and the report. Name the pdf file as "uniqname_AE584F19_Final_Project.pdf".